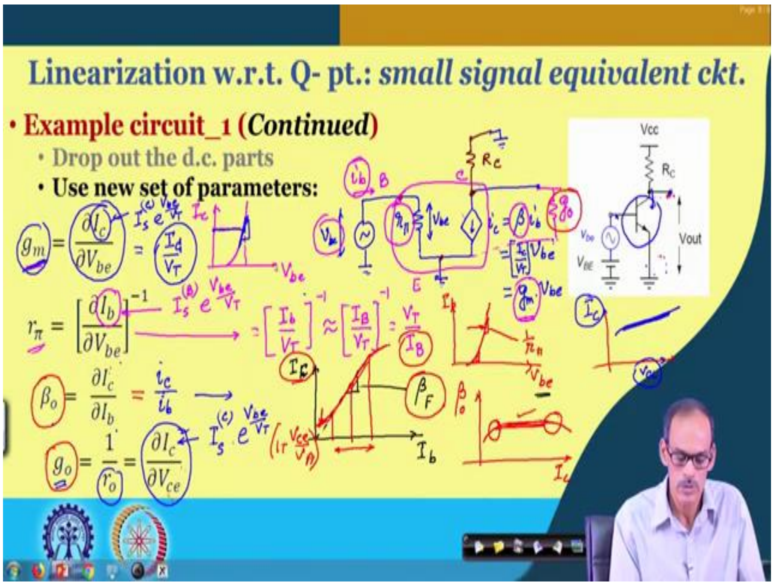
Analog Electronic Circuits

Prof. Pradip Mandal
Department of Electronics and Electrical Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture - 19
Linearization of Non – Linear Circuit Containing BJT (Contd.)

(Refer Slide Time: 00:27)



So, we are discussing that small signal equivalent circuit with respect to operating point which is basically linearization and we are talking about how do we. Once we have the circuit how we do linearize the circuit and so. So, whenever we are considering the equivalent small signal equivalent circuit, if I quickly draw the circuit we do have the small signal input and then base to emitter.

We have something called r_π and then we have the current source dependent current source. So, either we write in the form of voltage dependent or current dependent. So, this is i_c we may write in terms of i_b. In fact, there is another way of writing it you know in terms of v_{be}.

The voltage drop across the base to emitter terminal, we call this is v_{be} and incidentally since then there is no resistance here. We may say that this is also v_{be} and then. So, i_c it can be written in terms of v_{be} and we have seen that it is having some factor which is $\frac{I_C}{V_T}$ and this is referred as trans conductance parameter called $g_m \times v_{be}$.

Of course, we will come back to this point and then we do have the resistance R_C and this is connected to AC ground and of course, we obtain the voltage here. So, what we are saying is that we have seen graphical interpretation of linearization, we have seen in the form of equation and also we have seen in the form of equivalent circuit.

The equivalent circuit of the whole common emitter configuration involves the equivalent circuit of the transistor. So, if you see this part, this is the equivalent circuit of the transistor. So, this is the base terminal, this is the collector terminal and this is the emitter terminal.

In this course, we have different elements or parameters involved say for example, we already have discussed about the g_m. So, likewise we have the base to emitter resistance r_π and then we have from base to collector current gain and this is different from β_F this is actually referred as small signal current gain. And so far we are ignoring the early voltage in case if we consider the early voltage depending on the voltage at the collector. So, it will be having some variation of the collector current.

Namely based on the V_{ce} variation we may be having some variation in the collector current. So, if you consider that part also then across this current source. So, you may say that it will be having some conducting element. So, across this one there will be another conducting element it is actually it is going to the emitter.

So, this conducting element it is referred as output conductance in this case as you can say that this we refer as output. So, that is why output it is referred as output conductance and that is the meaning of each of these terms involve with the this equivalent circuit. So, if you see here, they do have of course, their own unambiguous definition.

Whenever you are talking about say g_m which is referred as transconductance of the device. So, how is it getting defined? This transconductance is representing the relationship between the collector current and V_{be}. So, you may recall this collector current I_c versus V_{be}. So, this characteristic curve is exponential with respect to some dc point or Q-point we are linearizing it and this linearization is basically a linear segment of the line and that line can be represented by the slope of the line.

And this slope of the line is nothing, but the first derivative of this current with respect to V_{be}. So, the transconductance its definition it is change in collector current with respect to change in V_{be} voltage. Note that this is I_c, this is V_{be}. So, if we vary this base to emitter voltage whatever the variation of this current we are observing keeping rest of the things constant, this relationship it is getting represented by this g_m trans conductance.

Since I am keeping other parameter constant particularly the collector to emitter voltage. So, that is why we are considering this is partial derivative and we already have in different way we have obtained this g_m. Expression of the g_m is $\frac{I_c}{V_T}$. In fact, the expression of this I_c in the form of a $I_S^{(C)} \times e^{\frac{V_{be}}{V_T}}$ and then if you take the derivative of first derivative of this expression what you can get is you will be getting this is $\frac{I_c}{V_T}$.

And this I_c of course, it is instantaneous I_c which is the slope of course, it depends on the instantaneous value, but then if we restrict our discussion within a small range then we can say that this I_c can be well approximated by I_C namely whatever the current we do have here at the DC operating point.

If you consider the slope of this line probably throughout this one we can assume that slope is remaining constant and hence you may say this is $\frac{I_C}{V_T}$. So, that is we obtain here the expression of the g_m is $\frac{I_C}{V_T}$. On the otherhand the current flowing through circuit in the small signal equivalent circuit if I say that this current is flowing through base to emitter terminal or rather base terminal say i_b. So, if I vary this voltage this current is changing.

So, you may say that if I take ratio of these two, then we may say that yes, it is basically base to emitter terminal conductance. So, if I see the conductance here namely if I take the if I observe the variation of the base terminal current with respect to V_{be} then equation wise $\frac{\partial I_b}{\partial V_{be}}$. So, that is the input port or base to emitter port conductance, if I take reciprocal of that that represents the base to emitter resistance its r_π.

Later I will see why we do have different subscripts, but at least we understand that the unit here it is ohm. So, we call this is base to emitter resistance. And again if you use a expression of this I_b in the form of $I_S^{(B)} \times e^{\frac{V_{be}}{V_T}}$ and then this part the internal part it becomes this is $\frac{I_b}{V_T}$ and then we may approximate that this I_b it is with respect to the Q-point. So, we can say that this is $\left[\frac{I_B}{V_T}\right]^{-1}$.

So, depending on this I_B of course, this value of this resistance it will vary. So, in fact, it is intuitive if you see whenever you are observing the let me use a different color. So, whenever we are observing the I_b versus V_{be} characteristic curve. So, depending on the operating point here we may get some slope here. So, that gives us one by r_π at that point. Now, if I come somewhere some other point here of course, their corresponding slope here it will be different so; obviously, at this point the corresponding r_π it will be different.

But as I said that if we restrict our discussion or from our experiment with respect to some operating point then you may say that this r_π it is remaining constant. So, based on the at the dc current there I_B we can say its resistance is $\frac{V_T}{I_B}$.

So, likewise the other parameter if you see here namely the β_o, β_o of course, you may say that sometimes we assume this β_o it is quote and unquote close to or equal to β_F. But strictly speaking it is having its own definition and normally we use β_o is the symbol which represents the relationship between the collector current variation with respect to base current variation.

So, obviously, this is different from $\frac{I_c}{I_b}$. In fact, normally what it is; what it is used here instead of writing here $\frac{\partial I_c}{\partial I_b}$. The relationship it is fairly linear, but whatever it is normally written in this form of $\frac{i_c}{i_b}$. And I like to say here one thing that, if the β_F it is remaining constant for whatever the range of this i_c and i_b we are considering then we may say that β_o it is same as β_F.

But in case if the β_F it is different then; obviously, depending on the operating point their dependency it will be different. So, ideally if we plot say I_c versus I_b. So, I_c along the y-axis and this is along the x-axis is the I_b. So, if you see both of them are having exponential relationship and so it is expected that if this both are exponential both are having exponential dependency on the same parameter called V_{be} then; obviously, it is expected to be it will be linear and slope of this line of course, it will be the β_F.

But in practical case actually if you go to lower level of current it is having a different slope. Here, it may be remaining constant and then again if you go to higher current again it will be having a bend. I will not be going detail of the sources of those in non-linear part, but as long as if you keep the transistor within certain range or if I say that if I consider range of this collector current within sake few 100's of μA to its maybe 10 mA for normal transistor you may say that this range it is fairly constant.

So, if we are within this range. So, wherever we consider slope remains constant and this if I say that, if I extend it if it is since it is going through the origin then the large signal β_F and β_o they are same. But if you are say somewhere here; obviously, the slope here it will be different which is the β_o it will be smaller than this one likewise if you consider the upper side.

If we sketch the variation of the β by that β_F or β_o with respect to say I_c what you can say that in the middle range you may say that it is remaining fairly constant. If you go a very low current it may drop and likewise if you go to a higher current it may drop.

This is the violation of low level injection and this is where the recombination current starts also becoming prominent. Somewhere in this range you may say that low level injection is valid as well as the recombination at the base region it can be ignored. And hence both of these curves are having quote and unquote same kind of relationship and hence the β is remaining constant.

That is about β. Now, if you consider other parameter particularly the conductance part. This conductance is due to the early voltage or you may say if you consider this circuit if I vary this collector voltage namely V_{ce} voltage due to early voltage.

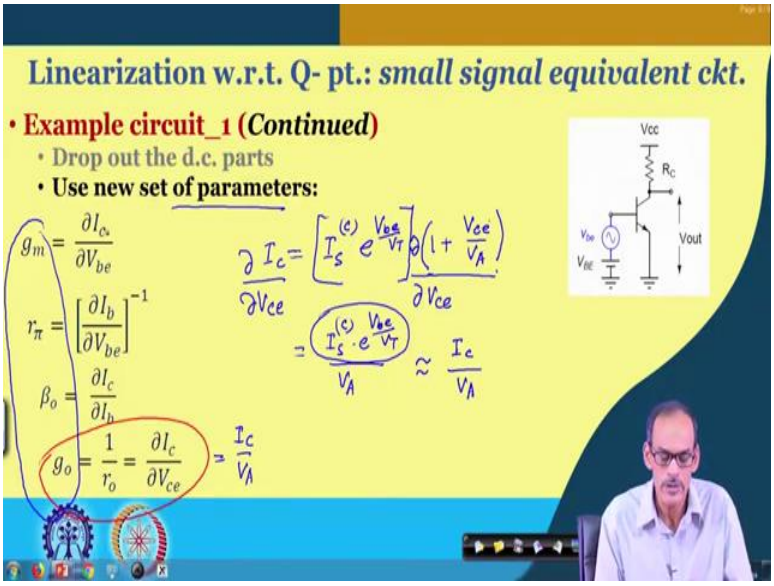
Due to early voltage effect, this I_c is having some dependency on the V_{ce}. In other words if we observe the collector current while varying the V_{ce} it is having a increment of the current. In other words if I increase this voltage, the total current is increasing which means that the total current it is not just completely defined by the condition at the base.

In fact, some part it is also dependent on this one. And since where if we change this voltage and if you are observing the same terminal current, you may say that if I take the ratio namely change in the current with respect to changing this voltage that gives nothing, but the output conductance.

So, that is why we are saying here the output conductance g_o is defined by change in the collector terminal current versus the collector to emitter voltage. And sometimes either we put this element in the form of conductance or some people are writing in the form of output resistance. So, if I take reciprocal of this one that gives us the output resistance or rather $\frac{1}{\text{output reesistance}}$ is defined by $\frac{\partial I_c}{\partial V_{ce}}$.

So, in fact, if you see again to get the expression of this part you require the expression of this I_c and so $I_S^{(C)} \times e^{\frac{V_{be}}{V_T}}$. Now, I have to consider this additional factor namely $\left(1 + \frac{V_{ce}}{V_A}\right)$ part then only I will be getting this part is non-zero.

(Refer Slide Time: 19:35)



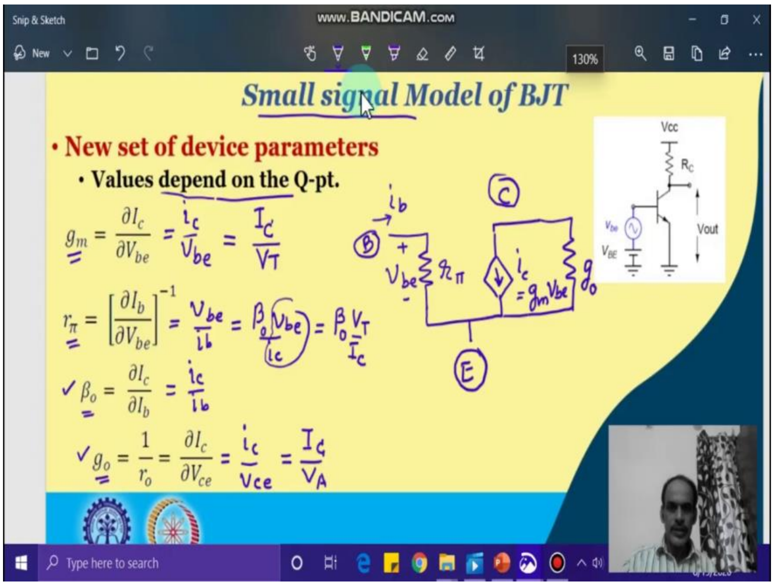
We have to take the expression of the collector current I_c which is having $I_S^{(C)}\ e^{\frac{V_{be}}{V_T}} \times \left(1 + \frac{V_{ce}}{V_A}\right)$.

Now, if I take the partial derivative with respect to V_{ce} and then of course, since we are changing only this one not this one. So, you may say that this part it is remaining constant, only this part it will be changing. So, if I say that if I take the derivative of the whole thing this part will be taking out and then we do have $\frac{\partial(\text{this part})}{\partial V_{ce}}$ and that gives us and this $I_S^{(C)} \times e^{\frac{V_{be}}{V_T}}$.

Now, we may approximate this whole thing by the collector current I_c. So, whole thing of course, collector current is having this part, but since this part is appearing as this. So, we may say that this part is very small compared to 1, only while you are taking derivative we are considering this part. And then this part it is I_c and this is V_A. So, the expression of g_o it is basically $\frac{I_c}{V_A}$.

So, we can say that all these parameters new set of parameters they are dependent on the operating point and as the operating point we are trying to keep it constant. So, we may say that for small signal equivalent circuit this parameters you may say that quote and unquote remaining unchanged. So, whenever we will be drawing the small signal equivalent circuit first step of course, we need to find this parameter and to get that first thing is that we have to find the operating point. So, let us see what it can summarize here.

(Refer Slide Time: 22:07)



We understand from our previous discussion that, whenever we are linearizing a non-linear circuit, we are essentially drawing small signal equivalent circuit. And while you are drawing the small signal equivalent circuit of an amplifier we are replacing BJT by its small signal equivalent model and this model involves a certain set of parameter device, we call it is device parameters.

And they are given here, as you can see here we do have transconductance defined by small signal collector current divided by small signal v_{be} and then the base to emitter impedance. Which is defined by $\frac{v_{be}}{i_b}$, small signal base current base terminal current then current gain small signal current gain which is defined by $\frac{i_c}{i_b}$.

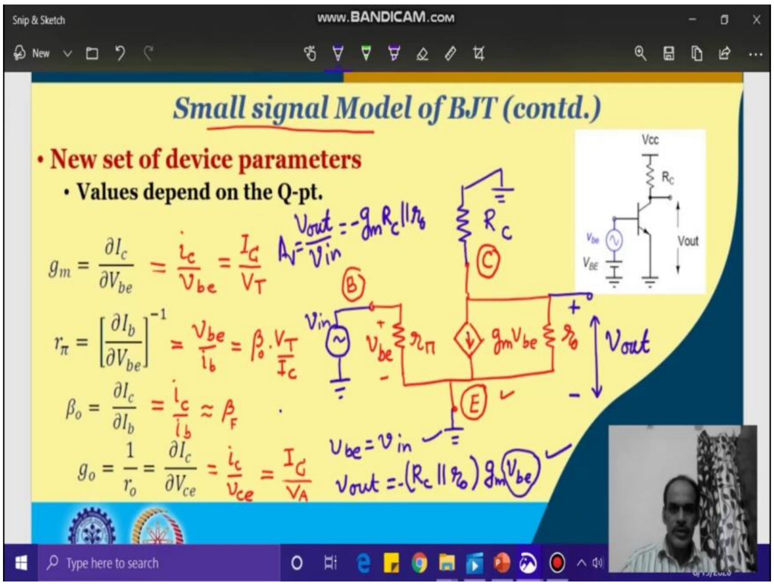
Moreover, output conductance is $\frac{1}{r_o}$ here, r_o is the output resistance. So, that is defined by $\frac{i_c}{v_{ce}}$. So, all these parameters as we have discussed that they depends on the operating point or to be more precise, if we express this small signal parameters in terms of the operating point. So, the g_m it is $\frac{I_C}{V_T}$; I_C collector current at the operating point and V_T is the thermal equivalent voltage.

So, likewise when you consider r_π and since it is $\frac{v_{be}}{i_b}$, so, it can be written in the form of i_c. So, we can replace i_b by $\frac{\beta_0}{i_c}$ here and then we do have v_{be} and this part it is the $\frac{1}{g_m}$. So, the expression of g_m we can write here or we can see that the expression of $r_\pi = \frac{\beta_0 V_T}{I_C}$. Where, β_0 is the small signal current gain. Likewise, when you consider g_0 it can be expressed in terms of once again it is operating point namely $\frac{I_C}{V_A}$, here V_A is the early voltage which is the device parameter.

Therefore, once we have the small signal parameters then small signal equivalent circuit can be obtained. Namely, base to emitter resistance r_π and the current entering to the base is i_b, voltage drop across this r_π it is v_{be}. This v_{be} on the other hand it is producing a current at the collector terminal. Collector to emitter we do have voltage dependent current source which is $i_c = g_m v_{be}$ and then also from collector to emitter, we do have finite conductance g_0. So, this is what the small signal equivalent circuit.

Now, this small signal equivalent circuit it would be very useful while we are converting the actual circuit into in terms of its small signal equivalent circuit. Now, let see how it can be deployed or what is its application, so, let see what its validity is.

(Refer Slide Time: 27:45)



Now here we do have the small signal equivalent circuit of the BJT and let us try to see that how it can be utilized to get the gain of the circuit. So, along with the small signal equivalent circuit of the BJT let you also integrate the other components of the circuit.

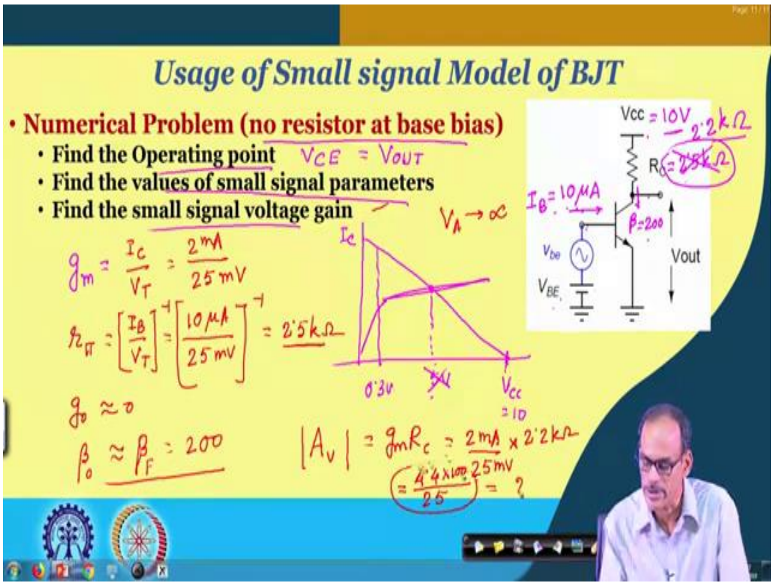
Namely, the R_C and then instead of connecting this to V_{cc} let you consider this is AC ground. Likewise, at the input, we do have an input signal, which is applied at the base with respect to a DC voltage, but for small signal, we consider the DC voltage it is zero. So, this is v_{in} small signal input and then at the emitter of course, it is connected to ground and whatever the voltage will be getting here with respect to the ground. So, that you call it is output and whatever the output you are getting here it is the small signal or the signal output.

So, if we analyze this simple circuit we can find the expression of v_{out} in terms of v_{in}. Incidentally, this v_{be} of the circuit is equal to v_{in}. So, the output voltage if you see this circuit v_{out} it is the an output resistance R_C which is coming in parallel with r_0 of course, r_0 it is $\frac{1}{g_m}$.

So, this multiplied by the current and the current flowing to this node it is $g_m v_{be}$ and if you see the polarity of the current and if I considered this is the polarity of the developed voltage this is plus and this is minus. Then we will be getting a – sign here.

Of course, this v_{be} it is v_{in}. So, by combining this equation and this equation, what we are getting here it is the expression of v_{out}. The expression of $\frac{v_{out}}{v_{in}} = -\, g_m \times (R_C \,/\!/\, r_0)$. This is nothing but the voltage gain of the circuit A_V. So, that is how the utilization of the small signal equivalent circuit. So, let us move to one numerical example to further illustrate on this.

(Refer Slide Time: 30:57)



In this numerical example if you see we have the same basic circuit, the main thing is at this bias we do not have any resistance. So, directly the voltage you are applying both the bias as well as the signal to the base. So, that makes this circuit simple and then we have say supply voltage say 10 V and let you consider this V_{BE} such that the current here it is say 10 µA. So, I am giving this V_{BE} in term such that the I_B it is 10 µA and then say β of the transistor it is say 200.

And so that gives us the quiescent current here it is 2 mA. Now, to get a good linearity range here namely to get good linearity range here we like to keep the operating point will away from this point as well as whatever the saturation limit or active region limit.

So, this is may be the maximum voltage you can have here it is a 10 V. So, load line it is intersecting there it its V_{cc} which is 10 V. This voltage it is very small, you may approximate that this is 0.3 V. So, roughly you even if you are keeping this voltage say at 5 V we do have good linearity range of this output I-V characteristic. So, if we do have 2 mA of DC current is flowing for this base current and this β then we may say that we may prefer to have 5 V; roughly 5 V drop across this R_c. So, that gives us this resistance maybe 2.5 kΩ.

So, suppose this information's are given. So, what I have done here it is. In fact, there may be two ways of framing the numerical problem, either we give this information and then directly we can ask draw the small signal equivalent circuit, find the values of the small signal parameters and then find the small signal voltage gain. Or probably we can give this bias condition such that this is 10 μA, this may be given to us then find this operating point and then we can go into this one, but whatever it is just for your practice instead of say let you consider instead of 2.5 which is giving us exactly 5 V.

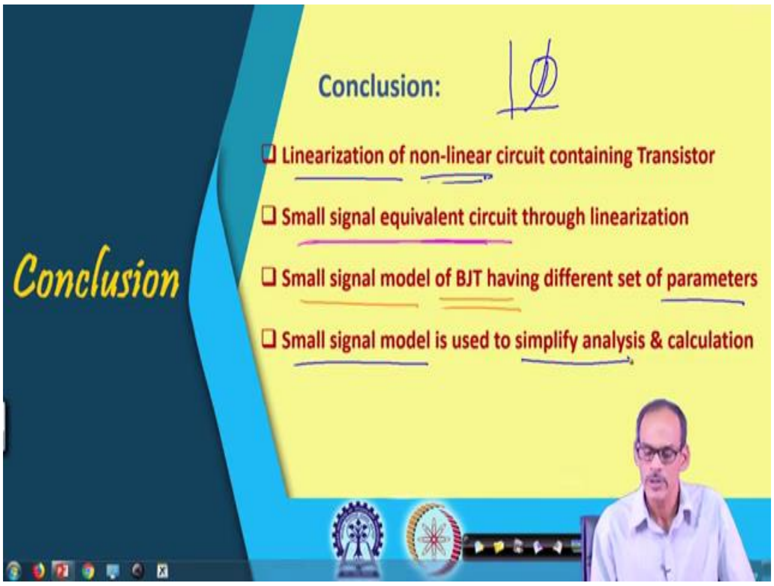
Let me consider this is something different may be 2.2 kΩ which is a practical value, 2.5 normally we do not get. So, probably you can try out find the operating point namely the I_B is given, collector current is also very straightforward. Then you can find the V_{CE} voltage which is nothing but V_{OUT} also. And then you can find the value of the small signal parameter g_m considering the early not early the thermal equivalent voltage to be 25 mV.

So, this is $\frac{I_C}{V_T}$. So, this is $\frac{2\text{ mA}}{25\text{ mV}}$. Then the r_π it is not required. So, still you can write r_π equals to $\frac{I_B}{V_T}$. So, this is 10 μA divided by sorry this is reciprocal of that. So, $\left[\frac{10\text{ μA}}{25\text{ mV}}\right]^{-1}$ so, that gives us 2.5 kΩ.

By the way the base to emitter resistance where you can see that it is much higher than typical diode on resistance, mainly because the base current though it is having exponential dependency the base current is very small. And then we may assume that early voltage it is very high. So, you may say that output conductance it is 0 and then β_o you can approximate to be β_F which is 200.

So, from that you can find what will be the small signal voltage gain, the small signal voltage gain it is if I consider magnitude it is $g_m \times R_c$ and $g_m \times R_c$ is $\frac{2\text{ mA}}{25\text{ mV}} \times 2.2\text{ k}\Omega$. So, this part it is 4.4; $\frac{4.4\text{ V}}{25\text{ mV}}$. So, that gives us $\frac{4.4}{25} \times 1000$ right. So, whatever the value it is coming.

(Refer Slide Time: 37:24)



We have observed that linearization of non-linear circuit containing some transistor is important. We are looking for linear behavior, then you may say that why are you looking for non-linear circuit; learning non-linear circuit gives us possibility of amplification. So, the non-linear behavior says like exponential behavior, it gives us highly good gain.

In fact, this is one of the sources of one of the factor which is giving us good gain. So, we require non-linear circuit, but then we are looking for linear circuit. So, that the in output signal nature it will be same as the input signal nature and then what we have done is that since you are looking for the linearization through that we are getting something called small signal equivalent circuit.

Which is basically linearization of the total circuit, keeping our constraint namely with progress of time dc wise the operating point should not change only the signal part it will change with time and also the average of the signal should be 0. So, that the operating point or the quiescent point should not change. So, that gives us some different notion called small signal equivalent circuit. Now, whenever you are drawing some small signal equivalent circuit of a given circuit the same notion it can be deployed for the simple transistor also.

So, whatever if I consider BJT in this case if I restrict our signal within some range then the BJT transistor can be represented by a linearized form which is referred as small signal model of the transistor. And why do you go for small signal model of the transistor? That simplifies the analysis.

We have seen model parameter it depends on the operating point. So, it is having different set of parameter, but once we get the value of those parameters and if we know that quiescent point is not changing then we can make use of those parameter value and then small signal model of the BJT or small signal equivalent circuit of the complete circuit it will be basically simplifying the analysis and calculation.

This is the heart of analog circuit. So, today we have discussed primarily on BJT. So, similar kind of discussion it will be there in our next class on MOSFET.